PARAMETRIC CURVE

The Parametric representation for Curves is :-

$$x = x(t)$$
$$y = y(t)$$
$$z = z(t)$$

A Curve is approximated by a piecewise polynomial Curve instead of piece linear Curve

Piecewise linear Curve

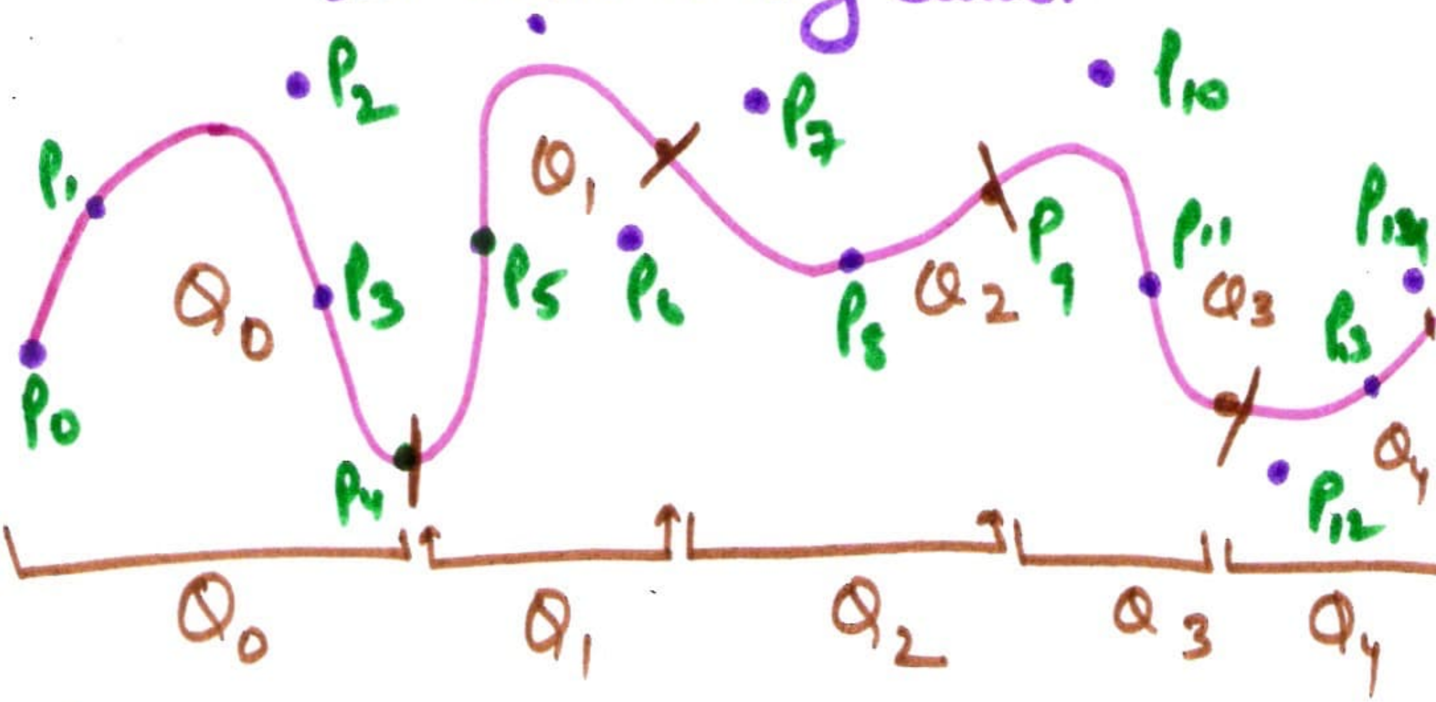
by Polylines & using linear Equation

Piecewise polynomial Curve.

Represented by Polynomial Equation.

For drawing Curve we need to specify some points through which it may or maynot completely follow:-

let take a big Curve:-



Q_0, Q_1, Q_2, Q_3 & Q_4 are the sections or segment of big curve. & P's are sample or control points

Each segment Q of the overall curves is given by three 3 functions x, y, z which are cubic polynomials in the parameter t or u.

Cubic means here is that the polynomial Eq. which is used to represent the curve is has degree of 3

The Cubic polynomials that define a curve segment

$$Q(t) = \begin{bmatrix} x(t) & y(t) & z(t) \end{bmatrix}$$

$$\left.\begin{aligned} x(t) &= a_x t^3 + b_x t^2 + c_x t + d_x \\ y(t) &= a_y t^3 + b_y t^2 + c_y t + d_y \\ z(t) &= a_z t^3 + b_z t^2 + c_z t \cdot d_z \end{aligned}\right\} \quad (1)$$

$$0 \leq t \leq 1$$

where $T = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix}$

The Coefficient matrix is defined as

$$C = \begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \\ d_x & d_y & d_z \end{bmatrix} \quad - (2)$$

So we can Rewrite equ. 1 as

$$Q(t) = [x(t) \;\; y(t) \;\; z(t)] = T \cdot C \quad \text{— } (3)$$

In General:-

In this C can be further be divided

$$C = M \cdot G$$

where $M = [m_g]_{4 \times 4}$ & $G = [g_1 \; g_2 \; g_3 \; g_4]^t$

M is a 4x4 basis matrix and G is a four element Column Vector of geometric Constants. Called the geometric Vector.

So $Q(t) = T \cdot M \cdot G$.

The Curve is a weighted Sum of the elements of the geometry matrix.

The weights are each Cubic polynomials of t, and are called the blending functions:-

$$B = T \cdot M$$

is

$$\frac{d\,Q(t)}{d(t)} = Q'(t) = \left[\frac{d\,x(t)}{dt} \quad \frac{d\,y(t)}{dt} \quad \frac{d\,z(t)}{dt}\right]$$

$$= \frac{d\,T \cdot C}{dt}$$

$$= \left[3t^2 \quad 2t \quad 1 \quad 0\right] \cdot C$$

$$= \left[3a_x t^2 + 2b_x t + c_x \quad 3a_y t^2 + 2b_y t + c_y \quad 3a_z t^3 + 2b_z t + c_z\right]$$